Dynamical systems lacking a spectral gap

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Résumé

Our world is neither compact nor periodic. It is therefore natural to consider dynamical systems on unbounded domains, where typically there is no spectral gap. I will present a (simple) method for studying the generators of such systems where a spectral gap assumption is replaced with an estimate of the *Density of States* (DoS) near zero. There are two main applications:
1) Dissipative systems: when the generator is non-negative, an estimate of the DoS leads to a so-called "weak Poincaré inequality" (WPI). This in turn leads (in some cases) to an algebraic decay rate for the L^2 norm of the solution. For instance, in the case of the Laplacian (generator of the heat equation) the WPI is simply the Nash inequality which leads to the optimal decay rate of $t^{-d/4}$.

2) Conservative systems: when the generator is skew-adjoint, an estimate of the DoS leads to a uniform ergodic theorem on an appropriate subspace. Examples include the linear Schrödinger equation and incompressible flows in Euclidean space.
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