

Elliptic Problems in Smooth and Non Smooth Domains

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SUMMARY

We are interested here in questions related to the **regularity** of solutions of **elliptic** problems with **Dirichlet** or **Neumann** boundary condition (see ([1]). For the last 30 years, many works have been concerned with questions when Ω is a **Lipschitz domain**.

We give here some complements for the case of the **Laplacian** (see [3]), the **Bilaplacian** ([2],[6]) and the operator $\operatorname{div}(A\nabla)$ (see ([5]), when \mathbf{A} is a matrix or a function, and we extend this study to obtain other regularity results for domains having an adequate regularity.

Using the duality method, we will then revisit the work of Lions-Magenes [4], concerning the so-called **very weak solutions**, when the data are less regular. Thanks to the **interpolation theory**, it permits us to extend the classes of solutions and then to obtain new results of regularity.

Keywords: Elliptic problems, Lipschitz domains, regularity

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